Optimizacija portfelja kriptovaluta koristeći mjere rizika repa distribucije i varijancu

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Crypto portfolio optimization through lens of tail risk and variance measures*

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Abstract

The choice of an adequate risk measure in portfolio optimization depends to a large extent on the characteristics and dynamics of the underlying assets. For investors and asset managers, a range of potential market risks provides much-needed insights into the optimization of their portfolio of assets. Since this paper focuses on multiple risk measures, it presents the investors with a better insight into the potential magnitude of the risk they are faced with. Since the risk-reward optimization target can be adjusted for a broad choice of risk measures in this paper we will test the performance of the classical risk measure i.e. standard deviation versus a tail risk measure such as expected tail loss (ETL). Our goal is to find which of the two offers the better performance for a portfolio of cryptocurrencies and if the differences are statistically significant. The setup for our analysis is testing two optimization targets (MinVar and MinETL) on 10 portfolios of cryptocurrencies randomly chosen from a sample of 70 cryptocurrencies with the highest market capitalization.

Key words: *portfolio optimization, cryptocurrency, risk evaluation, investments*

JEL classification: *E49, G11, P45*

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1. Introduction

The introduction of Bitcoin protocol in 2008 and its operational launch in January 2009 created a revolution in the world of alternative financial investments giving life to a new concept of a decentralized transaction system. Bitcoin, a pioneer in decentralized finance, also offered other appealing features such as small fees and almost instantaneous transactions involving no "middle-man" in the sense of a central governing body or an intermediary. Therefore, our paper looks at the risk profile of these investments.

When choosing the optimal portfolio for a given investor, it is paramount to consider the risk vs. expected return trade-off. Portfolio optimization is an area of interest that has gained attention during the last decades. Financial services and products have grown in number and sophistication. Nowadays, most trades are executed by a computer; retail investors have access to robo-advisors or even by themselves with their smartphones or computers through an online broker like Interactive Brokers or Robinhood.

Technology also catalysed the emergence of new asset classes like cryptocurrencies and Exchange Traded Funds (ETFs). Global assets invested in ETFs grew from merely 203.4 billion USD in 2003 to 9.1 trillion USD in 2021. Through the innovations in Blockchain, the asset of cryptocurrencies was developed. As of, September 23rd, 2020, the total capitalization of the 5,884 cryptocurrencies is of 334.8 billion USD according to CoinGecko. Harry Markowitz got a powerful insight into the selection of an optimal portfolio for an investor's given risk aversion. In his paper "Portfolio Selection", Markowitz (1952) considers that investors maximize expected returns and perceive variance as undesirable. For the lack of a better metric, he proposed the use of variance as the risk measure of an asset. He describes the feasible efficient line where a set of efficient portfolios are obtained for different risk profiles along the efficient frontier. Along the efficient frontier investor can find portfolios that maximize returns for an additional unit of risk (maximum Sharpe ratio portfolio) and the Global Minimum Variance (GMV) portfolio. Diversification is the key idea behind Markowitz's idea of risk-reward trade-off. The diversification is set through the variance of assets and covariances between them in a portfolio.

With the development of information technologies and the Internet in the last two decades, work on already existing ideas of digital money is intensifying. Parallel to this, the focus of academia and industry has been placed on preserving the value of asset holdings to which our paper is devoted. As a result of that work, a document entitled "Bitcoin: A Peer-to-Peer Electronic Cash System" was presented in 2008, describing a new decentralized transaction system that does not involve intermediaries between the entities of interest. The Bitcoin protocol was released on 9 January 2009, thus creating the infrastructure for the first cryptocurrency – bitcoin. Cryptocurrencies are a type of novel digital asset that is very hard to link to a classical approach of tying intrinsic value to market fundamentals which is common in financial markets. Bitcoin technology intends to enable almost instantaneous execution of transactions, with negligible fees without intermediaries or a central body and has therefore attracted great attention. An important feature of the Bitcoin transaction protocol is its open-source license. Namely, all algorithms and solutions used in its construction are available through the platform for the collaboration of developers. Anyone with an interest could access the program code, study it, and work on the Bitcoin protocol. If the proposals are in the direction of improvement, the community will accept the changes and improve the protocol. However, this also means that the existing protocol was easy to replicate, change and adapt to some needs, create a new cryptocurrency with new properties, and release it to the public. Open source feature has also contributed to the growth of many young, technology companies that develop their business idea on the public distributed ledger technology. Practical implementation of blockchain technology on the one hand, and positive public reactions to the idea of decentralization, on the other hand, contributed to supply and demand thus a completely new primary cryptocurrency market emerged. On the one hand, there were innovative companies that financed their idea by issuing cryptocurrencies, and on the other hand, there were investors who wanted to invest in this idea based on blockchain. With the development of the primary market, the number of exchanges also increased, thus creating a new, independently sustainable, ecosystem of the primary and secondary cryptocurrency market.

For the time being, the cryptocurrency market is burdened heavily by the absence of any regulatory framework, which means it is subject to all sorts of manipulations. Furthermore, there is no quantitative approach to calculating an intrinsic/fundamental value that would serve as a price stabilizer. On the other hand, the cryptocurrency market, its complexity, and its entire infrastructure is continuously growing. Due to its availability, an ever greater number of investors are investing and trading in cryptocurrencies, creating the need for greater academic research into the subject. Our primary aim in this paper is to define investment opportunities, but also the most appropriate risk measure in cryptocurrency portfolio optimization. The main hypothesis of our paper is that even when using very different risk measures as optimization targets in a crypto portfolio setup it is extremely difficult to achieve better performance than the general cryptocurrency market.

The objectives of this paper are as follows: (I) Evaluate the potential risk for a portfolio of cryptocurrencies; (II) Focus on current research about portfolio optimization; (III) Evaluate and interpret the results for risk management purposes; (IV) Conduct out-of-sample backtesting of tail risk and variance measures to ensure the appropriateness for risk management; (V) Ensure that proper risk measures are utilized in portfolio optimization.

The paper consists of six sections. Following the Introduction, Section 2 provides a literature review on portfolio optimization involving cryptocurrencies. Section

3 gives an overview of the data and methodology used, while Section 4 presents the empirical results and provides economic interpretation of the findings. Section 5 discusses the obtained results and outlines the economic implications of our findings. Finally, Section 6 draws conclusions, formulates implications and gives recommendations for future research.

2. Literature review

Although in this paper we use a slightly different methodology from previous research, we present the research results involving the construction and optimization of the portfolio in the secondary cryptocurrency market. Adding cryptocurrencies to traditional financial assets is beneficial to portfolio performance (Chuen et al., 2017). The authors optimized a portfolio of ten cryptocurrencies along with traditional financial assets, consisting of stock indices, gold, and real estate market index. Their results for all the optimization goals indicate the benefit of adding cryptocurrencies to a portfolio made up of traditional assets. Trimborn et al. (2019) similarly introduced cryptocurrencies into a portfolio composed of traditional financial instruments. Optimization was carried out with and without liquidity limitation and portfolio performance was compared. All portfolios that included cryptocurrencies performed better than portfolios composed only of traditional assets. Liquidity-constrained portfolios of equities and cryptocurrencies produce better cumulative returns than non-restricted portfolios.

Evaluation of portfolio performance composed of cryptocurrencies and traditional assets was also conducted by Petukhina et al. (2021). The authors divided the existing optimization models into four strategies: return-oriented strategies, risk-oriented strategies, risk-return-oriented strategies, and combined strategies. The authors applied these models to portfolios composed of 55 cryptocurrencies and 16 variables represented by five types of traditional assets. The performance of all the portfolios indicated the benefits of including cryptocurrencies in a portfolio along with traditional assets. The same portfolios achieved a lower cumulative return in a case when the liquidity limits were raised.

The benefits of constructing a portfolio of traditional financial assets and cryptocurrencies during the negative stock market movements caused by the COVID-19 pandemic were examined by Conlon et al. (2020) and Goodell and Goutte (2021). Both studies concluded that cryptocurrencies do not represent a safe haven for the majority of international equity markets and that cryptocurrencies in general do not provide a diversification benefit during market downturns. Both studies point out that the cryptocurrency USDT, whose value is pegged to the US dollar, can still serve as a safe haven investment for all of the international indices examined during times of market turmoil. This finding is logical, and actually has nothing to do with cryptocurrencies since it is actually a cash position in the US dollar during stock market crashes.

The cryptocurrency market can be viewed as a standalone niche financial market and thus it is useful to examine the possibility of constructing an efficient portfolio composed solely of cryptocurrencies with different allocation objectives. Liu (2018), Brauneis and Mestel (2018) and Platanakis et al. (2018) investigate the benefits of this approach. The abovementioned authors form multiple portfolios with different optimization goals, namely: risk minimization, returns maximization, and return to risk ratio maximization. Their results run contrary to widespread expectations and logic. All of their findings show that no optimization strategy can outperform the portfolio performance of an equally weighted portfolio, and thus they conclude that such a portfolio is the best choice when creating and modelling a portfolio in the secondary cryptocurrency market.

Cryptocurrency portfolio optimization issues were also examined by Tomić (2020) and Čuljak et al. (2022). Tomić (2020) used six different optimization objectives while taking into account the significant systematic impact of Bitcoin on the dynamics of the entire secondary cryptocurrency market. His results suggest that by controlling the exposure to the Bitcoin factor, better overall portfolio performance can be achieved through higher returns and risk-reward ratio. Following a similar logic, Čuljak et al. (2021) identified and described the benefits of sectoral cryptocurrency portfolio optimization using six portfolio optimization targets. Their results show that portfolio strategies performed better if they include sectoral cryptocurrencies from the financial, exchange, and business services sectors. Their findings confirm the existence of the possibility of modelling and optimizing the portfolio in the secondary market composed only of cryptocurrencies.

In line with this research overview, it is evident that our paper: (a) Focuses on a portfolio risk-range evaluation of cryptocurrencies, and this may in turn (b) Enrich the knowledge about their return-risk contribution when added into a portfolio of traditional assets, (c) Improve the understanding of liquidity of the combined portfolios, (d) Give economic bounds on the evaluated risk measures during market movements, (e) Weighted optimization leads to capturing economic relevant risk, (f) Support the primary and secondary market investments, (g) Provide a framework for individual and integrated risk management at financial markets.

3. Methodology

In the presented research papers, the authors examined the possibility of portfolio optimization in the cryptocurrency market with different optimization objectives, but only on a single sample of potential portfolio constituents. This sort of methodology is desirable when one wants to highlight the possibilities of different optimization goals, but it is not adequate when one wants to find the risk measure that is best suited to the dynamics of the cryptocurrency market. In this paper, we will create 10 portfolios (N-1 … N-10) out of 20 randomly selected potential components from a population of 70 cryptocurrencies selected by market capitalization. After taking the top 70 cryptocurrencies sorted by market capitalization we appliee pseudo-randomness generation algorithms (PRNG) to randomly choose 20 components (out of 70) that created the portfolios. Being repeated ten times, this generation process has created ten portfolios composed of 20 random components out of the top 70 cryptocurrencies sorted by market capitalization. We will analyse the portfolio results for two optimization strategies that define the minimal risk from the potential efficient frontier. The main difference between the two analysed optimization strategies is that the first optimization strategy uses the standard deviation as a risk measure (STDEV), while the second optimization strategy uses Expected Tail Loss (ETL) as a risk measure. By using this approach, we aim to complement previous research by finding the risk potential (range) that is suitable for cryptocurrency portfolio optimization.

In our analysis we use publicly available daily price data (in US dollars), collected from the Coinmarketcap – CMC platform. We use the daily observations within the period of 25/01/2018 – 01/08/2019, which is a sample of 554 daily observations. We chose the particular observation period in order to measure the portfolio performance during a volatile market regime. Throughou 2018, the cryptocurrency market had a drop in total market capitalization of 77%, while throughoue 2019, it achieved a significant growth of total market capitalization by 106%. Considering such market swings, the observed period represents an extremely volatile regime.

Two portfolios with the following risk minimization optimization targets were formed: minimum variance (MinVar) and minimum ETL (MinETL). Taking into account the previous research by Briere et al. (2015), Chuen et al. (2017), and Goodell and Goutte (2021), as well as the absence of a normal distribution of returns, apart from the variance (standard deviation), we chose to use a more robust tail risk measure as a portfolio risk measure, namely ETL. Our ETL approach follows the methodology of Rockafellar and Uryasev (2000), Conlon et al. (2020), and Čuljak et al. (2022). The ETL confidence level is set at 95%. The optimization is performed out of the sample, with equal parameters for each optimization target. The initial assessment of parameters and portfolio weights was performed on a sample of the first 30 days. Since cryptocurrency market is very volatile, a rolling window monthly rebalance (K = 30 days) was chosen. For each period $k + 1$, portfolio returns are extracted with respect to the results of the allocation optimization in the previous k and $k + K$ moments.

3.1. Global mean variance

In its original form, Modern Portfolio Theory focuses on minimizing the variance of the portfolio for a given level of expected return within certain theoretical assumptions, which is why it is often referred to as the mean variance (M-V) model. If the limitation of the required rate of return is omitted from the model, the optimization of the portfolio leads to Global Minimum Variance Portfolio – GMV. The formulation for GVM is given by:

$$\begin{aligned} &\min_{w} \quad \sigma_p^2(w) = w^{\mathrm{T}}\hat{\Sigma}w \\ &s.t. \;\; \mathbf{1}_N^{\mathrm{T}} w = 1, \qquad w_i \geq 0, \qquad i = 1, \ldots, N \end{aligned} \tag{1}$$

where σ_p^2 is the variance of the portfolio, w = $(w_1, w_2, ..., w_N)^T$ are the weights of individual assets in the portfolio and $\hat{\Sigma}$ is the estimated covariance matrix of assets N and their returns T. Additional constraints that are used: $\mathbf{1}_N$ represents a (N×1) vector where all elements of the vector represent the portfolio weights and their sum must be 1 (full investment constraint), and there is no short selling.

3.2. ETL as a risk measure

The drawback of the GMV approach is the assumption of a Gaussian distribution of asset returns for which the parameters are estimated. Considering the results of the study by Briere et al. (2015) and Chuen et al. (2017), which show the presence of a heavy-tailed cryptocurrency return distribution, we use, similar to Čuljak et al. (2021) and Petukhina et al. (2021), an approach based on the ETL methodology by Rockafellar and Uryasev (2000). By using the ETL as a risk measure, Global Minimum Variance Portfolio (GMV) becomes the Global Minimum ETL model (GMETL).

The cumulative distribution function of a loss function $z = f(w, y)$ is given by:

$$\Psi(w,\zeta) = P\{y | f(w,y) \leq \zeta\} \tag{2}$$

where w is fixed decision vector (i.e. portfolio weights), ζ loss associated with that vector and y uncertainties (e.g. market variables) that influence the loss. For a given confidence level α, the Value at Risk (VaR_α) is given by:

$$VaR_\alpha(w) = min\{y | \Psi(w,\zeta) \geq \alpha\} \tag{3}$$

If $f(w, y)$ exceeds the VaR, then the expected value of the loss (ETL) is given by:

$$\mathrm{ETL}_\alpha(w) = \frac{1}{1-\alpha} \int_{y(w) \leq VaR_\alpha(w)} y f(y|w)\, dy \tag{4}$$

Adjusting to the optimization goal, with a confidence level of 95%, gives us.

$$\begin{aligned} &\min_{w} \quad \mathrm{ETL}_{\alpha}(w) \\ &s.t. \quad \mathbf{1}_N^{\mathrm{T}} w = 1, \qquad w_i \geq 0, \qquad i = 1, \dots, N \end{aligned} \tag{5}$$

4. Empirical data and analysis

Based on the obtained optimization results we present and interpret out-of-sample backtesting results for each of the implemented optimization objectives. The success of a particular strategy was estimated through performance measures that include the CRyptocurrency IndeX – CRIX as a benchmark for the crypto market over the observation period. CRIX index is a crypto market index that represents one of the first cryptocurrencies indices and it was created by a team from Humboldt University of Berlin led by prof. Wolfgang Karl Härdle (Trimborn and Härdle, 2018). The index is weighted by its components' market capitalization and its methodology also adjusts to the sometimes illiquid cryptocurrency market. Each quarter, the components are evaluated and rebalanced.

The results are presented and interpreted in two steps. First, the results are compared and interpreted at the level of the asset allocation model and compared with the CRIX index. Second, in order to find the range of risk measures for cryptocurrency portfolio optimization, the results are compared between portfolios that differ in targeted risk measures that were used for portfolio optimization.

Table 1 shows the results of performance measures for the MinVar optimization strategy. We use the realized portfolio return annual geometric average return $R_{Gi} = prod(1 + R_{d,i})^{\frac{scale}{n}} - 1$, where $R_{d,i}$ is the daily realized return of portfolio i for period t, n the total number of existing observations and $scale$ number of observations in one year 252. Standard deviation, VaR and ETL is expressed annually by applying the square root of time rule $risk_{a,i} = risk_{d,i} \times \sqrt{252}$, where $risk_{d,i}$ is the daily risk measure.

The first column shows the absolute and relative performance measures used with the corresponding notations. The next 10 columns show the results of implemented performance measures for 10 portfolios in accordance with the optimization strategy, and the last column shows the results of performance measures of the CRIX index (i.e. benchmark of the cryptocurrency market).

The first two rows of the tables show the parameters of the fitted regression line between the portfolio returns as a dependent variable and the CRIX index as an independent variable. All portfolios have negative beta, indicating that they were moving in the opposite direction of the general cryptocurrency market. From the aspect of systemic risk, it can be said that strategy is less volatile than the general cryptocurrency market. On the other hand, only 1 portfolio achieved higher average returns than the CRIX index, as indicated by regression alpha, meaning that, on average, portfolios achieve weaker returns than the CRIX index. The same is confirmed by the realized geometric and cumulative return of the portfolio, where only two portfolios achieved a higher annualized and cumulative return compared to the general market. Risk measures (std dev, VaR, ETL) also suggest the superiority of the CRIX index compared to the created portfolios, where the CRIX index achieved lower risk values, except for the N-3 and N-6 portfolios. In the same way, relative performance measures also consider the index as an appropriate expected market risk measure in the observed period. All values, except for the N-2 and N-8 portfolio where the Sharpe Ratio value is less negative relative to the CRIX index, indicate that the CRIX index, during the period used in this study, is a significantly better choice compared to the MinVar optimization strategy.

Table 1: Comparative presentation of the results of performance measures for the MinVar optimization strategy

Performance Metrics		Asset Allocation										
		10 portfolios of 20 randomly selected cryptocurrencies										Index
		N-1	N-2	N-3	N-4	N-5	N-6	N-7	N-8	N-9	N-10	CRIX
Beta	β_i	-0.09	-0.09	-0.19	-0.10	-0.10	-0.09	-0.06	-0.07	-0.07	-0.05	1.00
Annualized Alpha	$\alpha_{a,i}$	-0.23	0.02	-0.17	-0.24	-0.27	-0.13	-0.31	0.00	-0.37	-0.40	0.00
Annualized Return	$R_{G,i}$	-0.42	-0.17	-0.30	-0.41	-0.43	-0.27	-0.50	-0.17	-0.52	-0.53	-0.18
Cumulative Return	CY	0.32	0.69	0.48	0.33	0.32	0.52	0.24	0.68	0.22	0.21	0.66
Annualized Std Dev	$\sigma_{a,i}$	0.76	0.64	0.58	0.72	0.70	0.61	0.82	0.63	0.73	0.72	0.63
Annualized VaR	$VaR_{a,i}$	1.26	1.05	0.97	1.20	1.17	1.00	1.37	1.03	1.22	1.30	1.04
Annualized ETL	$ETL_{a,i}$	1.58	1.32	1.22	1.50	1.46	1.26	1.72	1.29	1.53	1.63	1.31
Worst Drawdown	WD	0.87	0.74	0.69	0.87	0.85	0.73	0.92	0.71	0.92	0.87	0.78
Sharpe Ratio	$SR_{d,i}$	-0.56	-0.26	-0.51	-0.58	-0.61	-0.45	-0.60	-0.28	-0.71	-0.74	-0.29
MSquared	M^2	-0.35	-0.17	-0.33	-0.37	-0.39	-0.29	-0.38	-0.17	-0.45	-0.47	-0.18
Jensen's Alpha	α_i	-0.44	-0.18	-0.31	-0.43	-0.45	-0.29	-0.51	-0.18	-0.53	-0.54	0.00
Information Ratio	IR	-0.23	0.02	-0.13	-0.23	-0.25	-0.09	-0.30	0.02	-0.33	-0.35	/

Source: Author's calculation

Table 2 presents the results of the MinETL optimization strategy, with the identical structure to Table 1. With regards to the CRIX index, the presented results are similar to the MinVar ones. All portfolios achieved a negative value of the beta parameter, but in the case of MinETL two portfolios (N-5 and N-7) on average achieve a higher return than the CRIX index as indicated alpha regression. Only one portfolio (N-7) achieved a significantly higher geometric and cumulative returns than the general market. Marginally, compared to N-5 and N-7 portfolio, but still CRIX index managed to achieve lower risk values in the observed period. Relative performance measures also consider the index as an appropriate expected market risk measure. Apart from the N-5 and N-7 portfolios where the MSquared value is less negative relative to the CRIX index, all values indicate the superiority of the CRIX index over randomly created portfolios optimized by the MinETL strategy, at the 95% confidence level.

Table 2: Comparative presentation of the results of performance measures for the MinETL optimization strategy

Performance Metrics		Asset Allocation										
		10 portfolios of 20 randomly selected cryptocurrencies										Index
		N-1	N-2	N-3	N-4	N-5	N-6	N-7	N-8	N-9	N-10	CRIX
Beta	β_i	-0.10	-0.10	-0.24	-0.14	-0.14	-0.11	-0.13	-0.09	-0.11	-0.09	1.00
Annualized Alpha	$\alpha_{a,i}$	-0.18	-0.01	-0.33	-0.39	0.09	-0.38	0.97	-0.15	-0.44	-0.48	0.00
Annualized Return	$R_{G,i}$	-0.40	-0.26	-0.45	-0.56	-0.21	-0.51	-0.02	-0.31	-0.57	-0.62	-0.18
Cumulative Return	CY	0.35	0.54	0.29	0.19	0.61	0.23	0.96	0.46	0.17	0.14	0.66
Annualized Std Dev	$\sigma_{a,i}$	0.79	0.77	0.63	0.78	0.81	0.68	1.35	0.65	0.73	0.77	0.63
Annualized VaR	$VaR_{a,i}$	1.31	1.26	1.07	1.32	1.32	1.15	2.17	1.09	1.24	1.37	1.04
Annualized ETL	$ETL_{a,i}$	1.64	1.59	1.33	1.65	1.66	1.43	2.73	1.36	1.55	1.72	1.31
Worst Drawdown	WD	0.89	0.85	0.85	0.93	0.89	0.85	0.95	0.82	0.90	0.90	0.78
Sharpe Ratio	SR	-0.50	-0.33	-0.71	-0.71	-0.27	-0.75	-0.01	-0.48	-0.78	-0.80	-0.29
MSquared	M^2	-0.32	-0.21	-0.45	-0.45	-0.17	-0.48	-0.01	-0.30	-0.50	-0.51	-0.18
Jensen's Alpha	α_i	-0.42	-0.28	-0.47	-0.58	-0.24	-0.53	-0.04	-0.33	-0.59	-0.63	0.00
Information Ratio	IR	-0.20	-0.07	-0.29	-0.35	-0.03	-0.33	0.11	-0.13	-0.38	-0.41	/

Source: Author's calculation

5. Results and discussion

The first two rows of the tables show the parameters of the fitted regression line between the portfolio returns as a dependent variable and the CRIX index as an independent variable. All portfolios have negative beta, indicating that they were moving in the opposite direction of the general cryptocurrency market. From the aspect of systemic risk, we can say that strategy is less volatile than the general cryptocurrency market. On the other hand, only 1 portfolio achieved higher average returns than the CRIX index, as indicated by regression alpha, meaning that, on average, portfolios achieve weaker returns than the CRIX index. The obtained results of our research in this paper support our main hypothesis that even when using very different risk measures as optimization targets in a crypto portfolio setup, it is extremely difficult to achieve better performance than in the general cryptocurrency market. In our research, we opted to use the CRyptocurrency IndeX – CRIX as the general crypto market benchmark index. Against the performance of the crypto market benchmark – CRIX, we tested two competing optimization strategies that define the minimal risk from the potential efficient frontier. One optimization strategy uses the variance/standard deviation as a risk measure, and its optimization target is the minimization of variance/standard deviation (MinVar). The other tested optimization strategy uses a tail risk measure - Expected Tail Loss (ETL) as a risk measure, and its optimization target is the minimization of Expected Tail Loss (MinETL).

Evaluating the obtained results and performance measures between the two implemented optimization strategies, namely MinVar and MinETL, we find several interesting points. There is no significant difference between the values of the beta regression coefficients, all values in both tables are negative, which indicates the opposite direction of movement concerning the general crypto market movement (represented by the CRIX index). The economic consequence of these results is that, in the analysed period, our random optimized portfolios under both risk measures tend to act as a contrarian investment compared to the general crypto market trends and movements.

Looking at the alpha regression coefficient there is a slight difference between the two strategies and surprisingly it is favoring the MinVar approach in the prevailing market situation. Similar conclusion regarding the MinVar approach, in the prevailing market situation, is derived when looking at the geometric and cumulative returns, which are somewhat higher for MinVar approach compared to MinETL. The economic consequence of the obtained results in this regard is that optimization under a simpler and widely better known risk metric – variance, yields higher annualized returns for the investor compared to the more complex approach based on optimization under the tail risk measure – expected tail loss.

In terms of risk, the MinETL strategy gives higher values of all risk measures, standard deviation, VaR, and ETL, which is surprising since it should, by its construct, focus on the extreme tails of the underlying distribution. Taking into account the lower annualized returns and the higher risk estimates relative performance measures favor the MinVar approach at the 95% confidence level. Under the MinVar optimization strategy, only three portfolios (N-1, N-5, and N-7) have achieved worse values of relative performance measures. The economic consequence of such performance is that investing under MinETL optimization target will result in higher reserves on average creating additional, and in this case, unnecessary, opportunity costs compared to the MinVar optimization target.

Taking into account the lower annualized returns and the higher risk estimates the relative performance measures also favour the MinVar approach. This finding is somewhat surprising but could be contributed to several factors such as the observation period chosen but also the level of confidence being 95%. Since ETL as a tail risk measure focuses on the tails of the distribution it is possible that its true performance should be measured at higher quantiles, such as 99% or higher probabilities.

When comparing the results of optimized random portfolios under both analyzed risk measures to the general crypto market movement, represented in our research by the CRIX index we see a clear domination of the CRIX index. CRIX index, compared to both the MinVar and MinETL optimized portfolios, achieves higher regression alpha, realized geometric and cumulative return. Risk measures also show the superiority of the CRIX index compared to the optimized random portfolios, where the CRIX index achieved lower risk values. Consequently, relative performance measures also favor the CRIX index in the observed period, which supports the previous findings. All measures (Sharpe ratio (1963), Jensen's Alpha (1968), MS2, and Information ratio indicate that the CRIX index, representing the general crypto market, provides a most likely / expected risk values in addition to the other considered measures compared to the two employed optimization strategies.

Our research contributes to the current literature on optimization in financial markets, market risk measures, and cryptocurrency markets in general. Our research findings, to some extent, go contrary to some of the previous findings in the field of risk measures and optimization since our results prefer variance/ standard deviation as the risk measure being optimized as opposed to a tail risk measure such as expected tail loss ETL (e.g., Žiković, 2008; Žiković and Pečarić, 2010; Žiković, 2011; Žiković et al., 2015). On the other hand, our results with regard to the performance of the CRIX index in the general cryptocurrency market support previous findings in this field (e.g., Trimborn and Härdle, 2018; Trimborn et al., 2019; Čuljak et al., 2022).

The findings suggest that since all of the obtained results and all the employed metrics show that randomly created portfolios, even with optimized weights, perform inferior to the general cryptocurrency market (represented by the CRIX index), i.e., optimized randomly selected portfolios cannot beat the general market movement, i.e., crypto market index.

6. Conclusion

Our research addresses the question of whether random optimized portfolios of cryptocurrencies, optimized under different risk measures can beat the general market movement and to evaluate the performance of random portfolios optimized under different risk measures. In order to perform our analysis, we created 10 portfolios out of 20 randomly selected potential components from a population of 70 cryptocurrencies selected by market capitalization. In our analysis we used publicly available daily price data (in US dollars), collected from the Coinmarketcap – CMC platform. We had daily observations within the period of 25/01/2018 – 01/08/2019, creating a sample of 554 daily observations. Portfolios were formed with the following risk minimization optimization targets: minimum variance (MinVar) and minimum ETL (MinETL), and their performance at a 95% confidence level was evaluated against the CRyptocurrency IndeX – CRIX, used as a general crypto market benchmark index. We implemented our research methodology in two steps. First, we compared individual optimization results from two portfolio-optimization objectives to the CRIX index representing the cryptocurrency market benchmark. In the second step, in order to determine a risk measure (optimization objective) that achieves overall better portfolio performance, we compared the difference in performance between portfolio optimization under different risk goals

When comparing optimized random portfolios to the CRIX index, we see a prevailing domination of the CRIX index. The CRIX index, compared to both the MinVar and MinETL optimized portfolios, achieves higher regression alpha, realized geometric and cumulative return. Risk measures also show the superiority of the CRIX index compared to the created portfolios, where the CRIX index achieved lower risk values. Since all of the obtained results show that the CRIX index (representing the general cryptocurrency market) performs much better than the randomly created portfolios with optimized weights, both in terms of risk and return, we can conclude that for the analyzed time period, data frequency and at 95% confidence level, cryptocurrency market index is superior in performance to optimized randomly selected portfolios of cryptocurrencies, better portfolio performance, we compared the difference in performance between portfolio optimization under different risk goals.

The main limitations of our research are: (I) the time period sample used since we used two years of daily data, (II) using only a single confidence level of 95% in our analysis since it does not show the whole range of probabilities and likely leads to underperformance of tail risk measure (ETL). Tail risk measures are not adapted to measuring risk at such “low” probability levels, being more suitable for more extreme levels such as probability levels higher than 99%. Another possible limitation and a definitively promising alley for future research is our choice of sampling population. We used the market capitalization of cryptocurrencies as a must to form the population from which the random samples were drawn but definitively out rules for choosing the initial population could be used to test whether there are other, more important factors driving the crypto market besides the market capitalization which could improve the performance of random portfolios.

For further research, it is important to look into initial conditions, i.e., consider the importance of the adequate initial selection of potential portfolio components. This could be achieved by selecting the cryptocurrencies based on the crypto sector the cryptocurrencies are coming from, their public adoption, technological complexity, usefulness, and additional/further risk measures and metrics.

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Optimizacija portfelja kriptovaluta koristeći mjere rizika repa distribucije i varijancu

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Sažetak

Odabir odgovarajuće mjere rizika u velikoj mjeri ovisi o karakteristikama i dinamici imovine u koju se ulaže. Za investitore i upravitelje imovinom, raspon potencijalnih tržišnih rizika pruža prijeko potrebni uvid u optimizaciju portfelja imovine kojom upravljaju. Budući da je fokus ovog rada uspješnost više mjera rizika, investitorima se daje bolji uvid u potencijalnu veličinu rizika s kojim su suočeni. Budući da se cilj optimizacije rizika i dobiti može prilagoditi širokom izboru mjera rizika, u ovom ćemo radu testirati uspješnost klasične mjere rizika, tj. varijance/standardne devijacije u odnosu na mjeru rizika repa distribucije kao što je npr. očekivani gubitak u repu distribucije (ETL). Naš cilj je pronaći koja od ove dvije mjere rizika nudi najbolje rezultate za portfelj kriptovaluta i jesu li razlike statistički značajne. Osnova naše analize je testiranje dva cilja optimizacije (MinVar i MinETL) na 10 portfelja kriptovaluta koji su nasumično odabrani iz uzorka od 70 kriptovaluta s najvećom tržišnom kapitalizacijom.

Ključne riječi: *optimizacija portfelja, kriptovalute, procjena rizika, investicije*

JEL klasifikacija: *E49, G11, P45*

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